

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

LOCK DOWN REVISION QUESTIONS

SECTION A (40 marks)

1. Solve the inequality

$$\frac{x(x+2)}{x-3} \leq x + 1 \quad (5$$

marks)

2. Show that the line $\frac{x-2}{2} = \frac{y-2}{-1} = \frac{z-3}{3}$

Is parallel to the plane $4x-y-3z=4$ and find the perpendicular distance of the line from the plane. (5 marks)

3. Solve the equation

$$2\tan x - 3\cot x = 1$$

$$\text{For } 0^\circ \leq x \leq 360^\circ \quad (5 \text{ marks})$$

4. Calculate the co-ordinates of the point of the intersection of the curve

$$\frac{x}{2} + \frac{6y}{2} = 5 \text{ and } 2x = y - 2 \quad (5$$

marks)

5. The tangent to the curve $y = 2x^2 + 3x + 2$ at the point $(-2, 11)$ is

perpendicular

to the line $2x = y + 7$. Find the value of a and b . (5 marks)

6. Evaluate $\int_0^{\pi} \sin^3 x \, dx$ (5 marks)

7. Given that φ is a root of the equation $\varphi^2 - 2\varphi + 3 = 0$ show that $\varphi^3 = \varphi - 6$ (5 marks)

8. A spherical balloon is being inflated by gas being pumped at the constant rate of 200cm³ per second. What is the rate of increase of the surface area of the balloon

when its radius is 100cm?

(5 marks)

SECTION B (60 MARKS)

9. (a) If $(x + 1)^2 = x^2 + 2x + 1$, $x^2 + 7x^3 + 6x^2 + 2x + 1$, find the value of

A and B. (5 marks)

- (b) Prove that, if the equations $x^2 + ax + b = 0$ and $x^2 + 2x - 3 = 0$ have a common root and neither a and b is zero, then

$$a = \frac{5b}{2(b+3)}$$
 (7 marks)

10. (a) Given that $x = \sqrt{\frac{3+4x^2}{4+x^2}}$ find $\frac{dx}{x}$ in the simplest form. (7 marks)

$$\frac{4+x^2}{x^2}$$

- (b) If $x = x^4 + 3x^2 + 2x - 8x + 25x = 2$

(7 marks)

11. (a) Given that $x = \cos \theta$, $\theta \neq \pi$, $\frac{dx}{d\theta} =$

$$\frac{2}{1+x} = 1 - i \frac{1}{2} \theta.$$

(6 marks)

- (b) The polynomial $P(x) = x^4 - 3x^3 + 7x^2 + 21x - 26$ has a root α . Find $P(\alpha)$.

(6 marks)

12. (a) A right circular cone with semi vertical angle θ is inscribed in a sphere of radius r , with its vertex and rim of its base on the surface of the sphere. Prove that its volume is $\frac{8}{3}\pi r^3 \sin^4 \theta \cos^2 \theta$. (6 marks)

- (b) If r is constant and θ varies, show that the limits within which this volume must lie is $0 < V < \frac{32\pi r^3}{81}$ (6 marks)

13. (a) In any triangle ABC, prove that $\frac{1}{2}(a-b) \cos \frac{C}{2} = \frac{1}{2}(a+b) \sin \frac{C}{2}$

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$$\frac{1}{2} \frac{a+b}{2}$$

(6 marks)

- (b) In a particular triangle the angle A is 51° and $b=3c$. Find the angle B to the nearest degree. The area of this triangle is 0.47m^2 . Find side a to three decimal places.

14. (a) The points A and B have position vector $i-2j+k$ and $2ijk$ respectively.

Given that $OH = \lambda OA + \mu OB$ and O, H, A, B are coplanar, find the values of λ and μ .

i, j, k are unit vectors

Ratio of λ to μ .

Write down the vector equation of the line, l , through A which is perpendicular to OA . Find the position vector of P , the point of intersection of l and OB . (12 marks)

15. (a) Determine the equation of the normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at a point $P(a \cos \theta, b \sin \theta)$. (6 marks)

- (b) If the normal at P meets the x -axis at A and the y -axis at B , find the locus of the midpoint of AB . (6 marks)

16. (a) Solve the differential equation $\frac{dy}{dx} = \frac{y}{x} + \frac{y^2}{x^2}(2x^2 + 2xy)$, given that $y = \frac{\pi}{2}$ when $x = 1$. (5 marks)

$y = \frac{\pi}{2}$ when $x = 1$ (5 marks)

- (b) The rate of decay of a radioactive substance is proportional to the amount A remaining at any time t . Initially the amount was $5A_0$ and if the time taken for the amount of substance to become $\frac{1}{2}A_0$ is 1, find A at that time.

Find the time taken for the amount remaining to be reduced to $\frac{1}{20}A_0$.

(7 marks)